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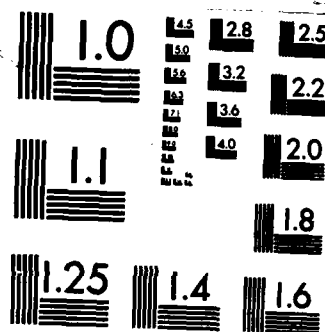
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LARGE-TIME BEHAVIOR OF VISCOUS
SURFACE WAVES

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LARGE-TIME BEHAVIOR OF VISCOUS SURFACE WAVES

J. Thomas Beale* and Takaaki Nishida**

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ABSTRACT

We consider the mathematical behavior of a viscous, incompressible fluid bounded above by an atmosphere of constant pressure and below by a horizontal bottom. After reviewing the existence and regularity theory for the equations governing the motion, we establish rates of decay for solutions near equilibrium. The function describing the height of the free surface decays like $t^{-1/2}$; the velocity field decays like t^{-1} . These estimates are shown first for the linearization about equilibrium and then for the full nonlinear problem. Complete details will be given elsewhere.

AMS (MOS) Subject Classifications: 68D05, 35Q10, 35R35

Key Words: viscous incompressible flow, surface waves, free surface

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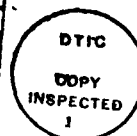
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SIGNIFICANCE AND EXPLANATION

One primary purpose of mathematical analysis is to describe the behavior of solutions of equations representing physical phenomena. The results shed light on the validity of the mathematical model as a representation of the physical process and may in turn offer new information about the phenomena. For the problem considered here, the motion near equilibrium of a viscous fluid bounded by a free surface, fundamental properties of existence and regularity were established by the first author in earlier work. This paper continues the study by describing further the behavior of solutions over a horizontal bottom. Solutions decay at a fixed rate because of the effect of viscosity in the interior of the fluid. The slow rate of decay is determined by the behavior of long wavelengths in the linearization about equilibrium.

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LARGE-TIME BEHAVIOR OF VISCOUS SURFACE WAVES

J. Thomas Beale* and Takaaki Nishida**

1. Introduction.

We are concerned with solutions global in time to a free surface problem of a viscous incompressible fluid, which is formulated as follows: The motion of the fluid is governed by the Navier-Stokes equations

$$(1.1) \quad \left. \begin{aligned} u_t + (u \cdot \nabla)u - \nu \Delta u + \nabla p &= 0 \\ \nabla \cdot u &= 0 \end{aligned} \right\} \quad \text{in } \Omega(t),$$

where $\Omega(t) = \{x \in \mathbb{R}^2, -b < y < \eta(t, x)\}$ is the domain occupied by the fluid. The free surface $S_F: y = \eta(t, x)$ satisfies the kinematic boundary condition

$$(1.2) \quad \eta_t + u_1 \eta_{x_1} + u_2 \eta_{x_2} - u_3 = 0 \quad \text{on } S_F.$$

The stress tensor satisfies the free boundary condition:

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$$(1.3) \quad p n_1 - \nu(u_{1,x_j} + u_{j,x_1}) n_j =$$

$$[g\eta - \beta \nu \{(1 + |\nabla \eta|^2)^{-1/2} \nabla \eta\}] n_1 \quad \text{on } S_F,$$

where n is the outward normal to S_F , g is the acceleration of gravity and β is the nondimensionalized coefficient of surface tension. On the bottom $S_B: y = -b$ we have the fixed boundary condition

$$(1.4) \quad u = 0 \quad \text{on } S_B.$$

We assume here that b is constant, although Theorem 1.1 below holds for $b = b(x)$.

We consider the initial value problem (1.1)-(1.4) with the data at $t = 0$

$$(1.5) \quad \begin{cases} \eta = \eta_0(x) & x \in \mathbb{R}^2, \\ u = u_0(x, y) & \text{in } \Omega_0, \end{cases}$$

where $\Omega_0 = \Omega(0)$. Local existence theorems for (1.1)-(1.5) are proved for both cases with or without considering surface tension ([1],[2]). The problem of existence global in time for (1.1)-(1.5) neglecting the surface tension ($\beta = 0$) has a difficulty which was pointed out in [1]. However if the surface tension is taken into account, the following global existence and regularity theorem is proved.

Theorem 1.1 ([2])

Let $3 < r < 7/2$. Assume the compatibility conditions on the initial data:

$$(1.6) \quad \begin{cases} \nabla \cdot u_0 = 0 & \text{in } \Omega_0, \\ \{((u_0)_1)_{,x_j} + (u_0)_j)_{,x_1} n_j\}_{\tan} = 0 & \text{on } y = \eta_0(x), \\ u_0 = 0 & \text{on } y = \eta_0(x), \end{cases}$$

There exists $\delta_0 > 0$ such that if the initial data satisfy

$$(1.7) \quad E_0 = |\eta_0|_{H^r(R^2)} + |u_0|_{H^{r-1/2}(\Omega_0)} < \delta_0,$$

then there exists a unique global solution η, u, p of (1.1)-(1.5), which satisfies

$$(1.8) \quad \eta \in \tilde{K}^{r+1/2}(R^+ \times R^2), u \in K^r(R^+ \times \Omega(t)), vp \in K^{r-2}(R^+ \times \Omega(t)).$$

Further, given any $T_1 > 0$ and any $k > 0$, there exists $\delta_1 > 0$ such that if

$$(1.9) \quad E_0 < \delta_1$$

then the solution becomes smooth for $t > T_1$, i.e.,

$$(1.10) \quad \begin{aligned} \eta &\in \tilde{K}^{r+k+1/2}((T_1, \infty) \times R^2), u \in K^{r+k}((T_1, \infty) \times \Omega(t)), \\ vp &\in K^{r+k-2}((T_1, \infty) \times \Omega(t)). \end{aligned}$$

In particular the solution with $k \geq 2$ is classical. Here $H^r(\cdot)$ is the usual Sobolev space with norm $|\cdot|_r$ on the domain $\cdot \cdot K^r((T_1, T_2) \times \Omega(t))$ is composed of the restriction to the fluid domain $\Omega(t)$ of the functions belonging to

$$(1.11) \quad K^r((T_1, T_2) \times \mathbb{R}^3) =$$

$$H^0((T_1, T_2); H^r(\mathbb{R}^3)) \cap H^{r/2}((T_1, T_2), H^0(\mathbb{R}^3)).$$

$\eta \in \tilde{K}^r(\mathbb{R}^+ \times \mathbb{R}^2)$ is defined as follows: $\eta \in K^r((0, T) \times \mathbb{R}^2)$ for any $T > 0$ and $\eta = \eta_1 + \eta_2$ such that $\eta_1 \in K^r(\mathbb{R}^+ \times \mathbb{R}^2)$ and η_2 is the Fourier transform in space-time of L^1 function of bounded support. See [2] for the details of the function spaces.

In this summary we give an asymptotic decay rate for the solution of the above theorem.

Theorem 1.2. If $\eta_0 \in L^1(\mathbb{R}^2)$, then there exists $\delta_2 > 0$ such that if

$$(1.12) \quad E_1 = E_0 + |\eta_0|_{L^1} < \delta_2,$$

then the solution has the decay rate:

$$|\partial^\alpha \eta(t)|_0 < CE_1(1+t)^{-(1+\alpha)/2}, \quad \alpha = 0, 1, 2$$

$$|u(t)|_2, |\nabla p(t)|_0 < CE_1(1+t)^{-1}.$$

In §2 we transform the free boundary problem (1.1)-(1.5) to an equivalent one on a fixed domain and reduce the components of the stress tensor to zero. The linear decay estimate is discussed in §3 and the nonlinear one in §4.

§2. Reduction of the Problem.

We recall some of the main ideas for the reduction of the free surface problem in [2]. First we use the transformation of the free boundary problem (1.1)-(1.5) to that on the fixed (equilibrium) domain: $\Omega = \{x \in \mathbb{R}^2, -b < y < 0\}$. Given $\eta(t, x)$ we extend it for $y < 0$ as follows:

$$(2.1) \quad \tilde{\eta}(t, x, y) = F^{-1}(e^{|t|y} \hat{\eta}(t, t)),$$

where $\hat{\eta}(t, t)$ is the Fourier transform with respect to x and F^{-1} is the inverse. If $\eta(t, \cdot)$ belongs to $H^s(S_F)$, then $\tilde{\eta}(t, \cdot, \cdot)$ belongs to $H^{s+1/2}(\Omega)$, where S_F now denotes the upper surface $y = 0$ of Ω . For each $t > 0$ we define the transformation θ on Ω onto $\Omega(t) = \{x \in \mathbb{R}^2, -b < y < \eta(t, x)\}$ by

$$(2.2) \quad \theta(x_1, x_2, y; t) = (x_1, x_2, \tilde{\eta} + y(1 + \tilde{\eta}/b)).$$

The vector field u on $\Omega(t) = \theta(\Omega)$ is defined from the vector field v on Ω by

$$(2.3) \quad u_i = \theta_{i,x_j} v_j / J = \alpha_{ij} v_j,$$

where J is the Jacobian determinant of $d\theta = (\theta_{i,x_j})$
 $= 1 + \tilde{\eta}/b + \tilde{\eta}_y(1 + y/b)$. This map preserves the property of
being divergence free, that is, $v \cdot v = 0$ in Ω iff $v \cdot u = 0$
in $\Omega(t)$.

Using the transformation (2.2), (2.3) and
 $u_{i,x_j} = \zeta_{1j} \theta_{i1}(\alpha_{ik} v_k)$, where $\zeta = (d\theta)^{-1}$ and so on, we can
rewrite the free surface problem (1.1)-(1.5) as one on the
equilibrium domain Ω as follows:

$$(2.4) \quad \eta_t - v_3 = 0 \quad \text{on } S_F,$$

$$(2.5) \quad \left. \begin{aligned} v_t - \nu \Delta v + vq &= F(\eta, v, vq) \\ v \cdot v &= 0 \end{aligned} \right\} \quad \text{in } \Omega,$$

$$(2.6)$$

$$(2.7) \quad v = 0 \quad \text{on } S_B,$$

$$(2.8) \quad \left. \begin{aligned} v_{1,x_3} + v_{3,x_1} &= F_1(\eta, v), \quad 1 = 1, 2 \end{aligned} \right\} \quad \text{on } S_F.$$

$$(2.9) \quad q - 2\nu v_{3,x_3} - (g - \beta\Delta)\eta = F_3(\eta, v)$$

Here we have gathered the linear terms on the left hand side and
all the nonlinear terms on the right hand side of the equation.

Next we reduce the tangential component of the stress tensor F_i , $i = 1, 2$ to zero: Given $F_i \in H^{r-3/2}(S_F)$, $i = 1, 2$, choose the vector $z \in H^{r+1}(\Omega)$ satisfying the condition

$$\begin{cases} z = 0, & \sigma_y z = 0, & \sigma_y^2 z = (F_2, -F_1, 0) & \text{on } S_F, \\ z = 0, & \sigma_y z = 0 & & \text{on } S_B. \end{cases}$$

Then $w = v \times z$ satisfies

$$\begin{aligned} w_3 &= 0, & w_{1,x_3} + w_{3,x_1} &= F_1, & i &= 1, 2 & \text{on } S_F, \\ v \cdot w &= 0 & & \text{in } \Omega, \\ w_3 &= 0 & & \text{on } S_B. \end{aligned}$$

Therefore η , $v' = v - w$, q satisfy the system (2.4)-(2.9) with the replacements F by $F_4 = F - w_t + \nu \Delta w$ and F_i , $i = 1, 2$ by 0. The prime in v' is omitted hereafter.

Finally we rewrite the system (2.4)-(2.9) with $F = F_4$, $F_i = 0$, $i = 1, 2$, for η, v, q in the operator form. Let P be the projection on the subspace of solenoidal vectors orthogonal to the subspace $Gr^0 = \{vP: v \in H^1(\Omega), v = 0 \text{ on } S_F\}$ of $H^0(\Omega)$, i.e.,

$$(2.10) \quad H^0 = PH^0 \oplus Gr^0.$$

Applying P to (2.5) we have

$$(2.11) \quad v_t - \nu P \Delta v + P v q = P F_4.$$

Here $P v q$ can be decomposed to three parts as follows:

$$p \nabla q = \nabla \pi^{(1)} + \nabla \pi^{(2)} + \nabla \pi^{(3)},$$

where $\pi^{(i)}$, $i = 1, 2, 3$, are defined by

$$\pi^{(1)} = 2\nu v_{3,x_3}, \quad \pi^{(2)} = g\eta - \beta \Delta \eta, \quad \pi^{(3)} = F_3 \quad \text{on } S_F,$$

$$(2.12) \quad \begin{aligned} \Delta \pi^{(1)} &= 0 & \text{in } \Omega, \\ \sigma_Y \pi^{(1)} &= 0 & \text{on } S_B. \end{aligned}$$

We define

$$A v = -P \Delta v + \nabla \pi^{(1)},$$

$$(2.13) \quad \begin{cases} R v = v_3|_{S_F}, \end{cases}$$

$$R^*((g - \beta \Delta)\eta) = \nabla \pi^{(2)}.$$

Using these notations the system (2.4), (2.11) has the form

$$(2.14) \quad \eta_t = R v,$$

$$(2.15) \quad v_t + A v + R^*((g - \beta \Delta)\eta) = f,$$

where $f(\eta, v, \nabla q) = P F_4 - \nabla \pi^{(3)}$. We regard (2.6), (2.7), (2.8) with $F_1 = 0$ as domain conditions on A .

3. Rates of Decay for the Linear Problem

We investigate the decay rate of the solution of the linearized equations

$$(3.1) \quad \eta_t = Ru, \quad$$

$$(3.2) \quad u_t + Au + R^*((g - \beta \Delta)\eta) = 0, \quad$$

$$(3.3) \quad \eta(0) = \eta_0, \quad u(0) = u_0 \quad \text{at } t = 0.$$

These are supplied with the conditions:

$$(3.4) \quad v \cdot u = 0 \quad \text{in } \Omega,$$

$$(3.5) \quad u_{1,x_3} + u_{3,x_1} = 0, \quad i = 1, 2 \quad \text{on } S_F,$$

$$(3.6) \quad u = 0 \quad \text{on } S_B.$$

Theorem 3.1. Let $E_2 = |\eta_0|_{L^1} + |\eta_0|_{5/2} + |u_0|_0$. Then the solution of (3.1)-(3.6) has the decay rate ($t \geq 1$):

$$(3.7) \quad |\partial^\alpha \eta(t)|_0 \leq C_0 E_2 t^{-(1+\alpha)/2}, \quad 0 \leq \alpha \leq 5/2.$$

$$|u(t)|_2 \leq C_0 E_2 t^{-1}.$$

The theorem is proved in several steps.

Let $\mathcal{X} = \{v = (\eta, u): \eta \in H^1(S_F), u \in PH^0(\Omega)\}$, where $(\rho, \eta)_1 = g(\rho, \eta)_0 + \beta(\nabla \rho, \nabla \eta)_0$ is the inner product of $H^1(S_F)$

and set $W = \{v: \eta \in H^{5/2}(S_F), u \in PH^2(\Omega) \text{ and } u \text{ satisfies (3.4), (3.5), (3.6)}\}$. Let us define the operator

$$(3.9) \quad G v = \begin{bmatrix} 0 & R \\ -R^*(g-\beta\Delta)\eta & -A \end{bmatrix} \begin{bmatrix} n \\ u \end{bmatrix} \quad \text{on } D(G) \subseteq \mathcal{K},$$

and consider its closed extension which will be denoted by G again.

Lemma 3.2. *The operator G generates a contraction semigroup e^{tG} on \mathcal{K} , and $W \subset D(G)$.*

Consider the resolvent equation:

$$(3.10) \quad (\lambda - G) \begin{bmatrix} \eta \\ u \end{bmatrix} = \begin{bmatrix} h \\ f \end{bmatrix}.$$

The resolvent of G can be extended in to the left half-plane, as shown in lemmas 3.3-3.5.

Lemma 3.3. *For any $\tau_0 > 0$ there exists $c_0 > 0$ such that if*

$$\lambda \in \{\lambda = \sigma + i\tau, -c_0|\tau| < \sigma < \tau_0, |\tau| > \tau_0\},$$

then the solution of (3.10) has the estimate

$$(3.11) \quad |u|_2 + |\lambda| |u|_0 + |\lambda|^{-1} |Ru|_{5/2} + |\eta|_{5/2} + |\lambda| |\eta|_{3/2} \\ \leq C(|f|_0 + |h|_{5/2}) .$$

We treat the resolvent near $\lambda = 0$ in two cases separately: (i) the supports of $\hat{h}(\xi)$, $\hat{f}(\xi, y)$ belong to $(|\xi| \geq \xi_0)$; (ii) the supports belong to $(|\xi| \leq \xi_0)$. Here $\hat{\cdot}$ means the Fourier transform with respect to x .

Lemma 3.4. For any $\xi_0 > 0$ there exists $r_0 > 0$ such that if $\lambda \in \{|\lambda| < r_0\}$ and the supports of $h(\xi)$, $f(\xi, y)$ belong to $(|\xi| \geq \xi_0)$, then the resolvent equation (3.10) has the solution (η, u) satisfying

$$(3.12) \quad |u|_2, |\eta|_{5/2} \leq C(|h|_{5/2} + |f|_0) .$$

Let $\hat{G}(\xi)$ be the Fourier transform of G with respect to x .

Lemma 3.5. There exist $\xi_1 > 0$ and r_1, r_2 , with $\nu(\pi/2b)^2 > r_2 > r_1 > 0$, such that if $r_1 < |\lambda| < r_2$, then $(\lambda - \hat{G}(\xi))^{-1}$ exists for $|\xi| < \xi_1$. For small ξ there is a one-dimensional eigenspace which is analytic with respect to ξ . The eigenvalue and eigenvector have the following expansions.

$$\begin{aligned}
\lambda &= -(gb^3/3\nu)|\xi|^2 + O(|\xi|^4), \\
\eta &= 1 + O(|\xi|^2), \\
(3.13) \quad \begin{cases} u_j = i(gt_j/2\nu)(y^2-b^2) + O(|\xi|^3), & j = 1, 2, \\ u_3 = (g|\xi|^2/2\nu)(y^3/3-b^2y-2b^3/3) + O(|\xi|^4). \end{cases}
\end{aligned}$$

By using lemmas 3.2-3.5 the decay estimate (3.7) can be proved by the transformation of the integral path of the representation

$$(3.14) \quad v(t) = e^{tG}v_0 = \lim_{\sigma \rightarrow \infty} \int_{\sigma-1r}^{\sigma+1r} e^{\lambda t}(\lambda-G)^{-1}v_0 \, d\lambda, \quad \sigma > 0$$

to the left half-plane.

14. Nonlinear Decay Estimates.

The free surface problem (1.1)-(1.5) was reduced to the following system in 12.

$$(4.1) \quad \eta_t - R u = 0,$$

$$(4.2) \quad u_t + A u + R^* ((g - \beta A)\eta) = f,$$

$$(4.3) \quad \eta(0) = \eta_0, \quad u(0) = u_0,$$

where f consists of nonlinear terms depending on η , u , vp and their derivatives.

The initial value problem (4.1)-(4.3) has a unique solution (Theorem 1.1) which becomes smooth for $t \geq T_1 > 0$. Namely we know that if $E_0 < \delta_1$, then

$$(4.4) \quad \begin{cases} \eta(t) \in C(0, \infty; H^{5/2}), \text{ i.e., } \tilde{\eta}(t) \in C(0, \infty; H^3), \\ u(t) \in C(0, \infty; H^2), \text{ and} \end{cases}$$

$$(4.5) \quad \tilde{\eta} \in \tilde{K}^7((1, \infty) \times \Omega), \quad u(t) \in K^6((1, \infty) \times \Omega).$$

Let us define

$$(4.6) \quad M(\eta, u; t) = \max_{\alpha=0,1,2} \left\{ \sup_{0 \leq s \leq t} (1+s)^{(1+\alpha)/2} |D^\alpha \tilde{\eta}(s)|_0, \right. \\ \left. \sup_{0 \leq s \leq t} (1+s)^{3/2} |D^3 \tilde{\eta}(s)|_0, \sup_{0 \leq s \leq t} (1+s) |u(s)|_2 \right\}.$$

$$E(\eta, u) = |\tilde{\eta}|_{\tilde{K}^7((1, \infty) \times \Omega)} + |u|_{K^6((1, \infty) \times \Omega)}.$$

Since the linearized equation is solved by the semigroup e^{tG} on X in 13, this solution satisfies the variation of constants formula:

$$(4.7) \quad v(t) = e^{tG} v_0 + \int_0^t e^{(t-s)G} f(s) ds,$$

where $v = \begin{pmatrix} \eta \\ u \end{pmatrix}$ and $f = \begin{pmatrix} 0 \\ f(\eta, u, v_p) \end{pmatrix}$.

The first term of (4.7) was already estimated in 13, i.e.,

$$(4.8) \quad \begin{bmatrix} \eta_0(t) \\ u_0(t) \end{bmatrix} = e^{tG} \begin{bmatrix} \eta_0 \\ u_0 \end{bmatrix} \text{ has the decay rate:} \\ |\partial^\alpha \eta_0(t)|_0 \leq C_0 E_2 t^{-(1+\alpha)/2}, \quad 0 \leq \alpha \leq 5/2, \text{ i.e.,}$$

$$(4.9) \quad \begin{aligned} |D^\alpha \tilde{\eta}_0(t)|_0 &\leq C_0 E_2 t^{-(1+\alpha)/2}, \quad \alpha = 0, 1, 2, \text{ and} \\ |D^3 \tilde{\eta}_0(t)|_0 &\leq C_0 E_2 t^{-3/2}, \\ |u_0(t)|_2 &\leq C_0 E_2 t^{-1}. \end{aligned}$$

It is sufficient by (4.4) to prove the decay rate for $t \geq 2$.

Let us decompose the second term of (4.7) into three parts

$$(4.10) \quad \int_0^t \{e^{(t-s)G} \begin{bmatrix} 0 \\ f(s) \end{bmatrix}\} ds = \int_0^{t/2} \{ \} ds + \int_{t/2}^{t-1} \{ \} ds + \int_{t-1}^t \{ \} ds \\ = \begin{bmatrix} \eta_1 \\ u_1 \end{bmatrix}(t) + \begin{bmatrix} \eta_2 \\ u_2 \end{bmatrix}(t) + \begin{bmatrix} \eta_3 \\ u_3 \end{bmatrix}(t).$$

Lemma 4.1. Let $E_3 = |\tilde{\eta}|_3 + |u|_2 + |Dp|_0$. We have the estimates:

$$(4.11) \quad \begin{aligned} |F|_0 &\leq C(E_3) ((|D\tilde{\eta}|_2 + |u|_2)(|D\tilde{\eta}|_2 + |u|_2 + |Dp|_0) \\ &\quad + |\tilde{\eta}|_{L^\infty}(|D^2 u|_0 + |Dp|_0)). \\ |Dp|_0 &\leq C(E_3) ((|D\tilde{\eta}|_2 + |u|_2)(|D\tilde{\eta}|_3 + |u|_3 + |Dp|_1) \\ &\quad + |\tilde{\eta}|_{L^\infty}(|D^3 u|_0 + |D^2 p|_0)). \\ |D^2 F|_0 &\leq C(E_3) ((|D\tilde{\eta}|_2 + |u|_2)(|D\tilde{\eta}|_4 + |u|_3 + |Dp|_1) \\ &\quad + (|\tilde{\eta}|_{L^\infty} + |D\tilde{\eta}|_{L^\infty} + |D^2 \tilde{\eta}|_1)(|D^4 u|_0 + |D^3 p|_0)). \end{aligned}$$

Let \tilde{F}_1 , $i = 1, 2, 3$ on Ω be the extension of F_1 on S_F by using $\tilde{\eta}$ for η . We have

$$(4.12) \quad |\tilde{F}_1|_0 \leq C(E_3)(|D\tilde{\eta}|_2 + |\tilde{\eta}|_{L^\infty})|u|_2$$

$D^\alpha \tilde{F}_1$, $\alpha = 1, 2, 3$, have the same estimate as $D^{\alpha-1} F$ as above. Therefore f has the same estimate as (4.11).

Lemma 4.2. For $t \geq 2$, we have for $i = 1, 2$

$$(4.13) \quad M(\eta_1, u_1; t) \leq C_1 \sup_{0 \leq s \leq t} (1+s)^{3/2} |f(s)|_0$$

$$\leq C_1 C(E_3) M(\eta, u; t)^2.$$

Also we have for $i = 3$

$$(4.14) \quad M(\eta_3, u_3; t) \leq C_2 \sup_{t-1 \leq s \leq t} (1+s)^{3/2} |f(s)|_2$$

$$\leq C_2 C(E_3) M(\eta, u; t) (M(\eta, u; t) + E(\eta, u)) .$$

This is proved by Theorem 3.1 and by Lemma 4.1 and Sobolev-Nirenberg's inequality [4]. In particular, we know that since by (4.5) $\tilde{\eta}(t)$ and $u(t)$ are bounded in H^6 and H^5 respectively for any $t \geq 1$, we have by (4.6) for any $t \geq 1$

$$|D^{3+\alpha} \tilde{\eta}(t)|_0 \leq C t^{(\alpha-3)/2}, \quad \alpha = 1, 2,$$

$$|D^{2+\alpha} u(t)|_0 \leq C t^{(\alpha/3-1)}, \quad \alpha = 1, 2.$$

Proof of Theorem 1.2. It follows from (4.9) and Lemma 4.2 that

$$\begin{aligned} M(\eta, u; t) \leq & C_0 E_2 + C_1 CM(\eta, u; t)^2 \\ & + C_2 CM(\eta, u; t) (M(\eta, u; t) + E(\eta, u)). \end{aligned}$$

Therefore there exists $\delta_2 > 0$ such that if $E_2 < \delta_2$, $E_0 < \delta_1$, then

$$M(\eta, u; t) \leq C E_2.$$

This proves Theorem 1.2.

Remark. If the fluid has an infinite depth, the eigenvalue of $G(\xi)$ has the following expansion:

$$(4.15) \quad \lambda(\xi) = \pm \sqrt{g|\xi|} - 2\nu|\xi|^2 - O(|\xi|^{11/4}),$$

which is quite different from (3.13). The details will be published elsewhere.

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